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UNITS AND SYSTEMS OF WEIGHTS AND MEASURES

This Letter Circular has been prepared to answer some of the questions frequently asked about units and systems of weights and measures. In this Letter Circular the expression "weights and measures" is used in its basic sense referring to measurements such as those of length, mass, and capacity, thus excluding such topics as units of electricity, thermometry, and photometry. This is neither a treatise nor an exhaustive text and is not intended to replace printed material on this subject. It is intended, rather, as a vehicle for presenting in simple language some of the many aspects of the general subject of weights and measures that need to be considered whenever specific questions relating to weights and measures are under discussion.

I. Origin and Early History of Units and Standards

1. Units and standards.- In order to avoid confusion and error, it is essential that there be established and kept in mind the distinction between the terms "units" and "standards".

A unit is a value, quantity, or magnitude in terms of which other values, quantities, or magnitudes are expressed. In general, it is fixed by definition and is independent of such physical conditions as temperature. Examples.- The yard, the pound, the gallon, the meter, the liter, the gram.

A standard is a physical embodiment of a unit. In general it is not independent of physical conditions, and it is a true embodiment of the unit only under specified conditions. For example, a yard standard has a length of one yard when at some single temperature and supported in a certain manner. If supported in a different manner, it would have to be at a different temperature in order to have a length of one yard.

2. General survey of early history of weights and measures.- The beginnings of the story of the development of weights and measures go back to primitive man in prehistoric times. Hence there is a great deal of uncertainty about the origin and early history of weights and measures. Many believe that the units first used by primitive man were those of length and weight and that units of area, volume, and capacity are of much later origin. Units of length may have been the earliest. These

were derived from the limbs of the body, and included the length of the human foot, the width of the palm, the length of the forearm, etc. Units of weight included weights of grain and weights of shells.

At first these units were not very definitely defined. Later they became somewhat more definite when, for example, the foot became the length of the foot of a tribal chief or other ruler. At a much later date physical standards were made and deposited for safekeeping in a temple or other place of security. These early physical standards were usually very crude; it is generally considered, however, that they were as satisfactory for the needs of the people at that time as our most modern standards are for our own needs.

Our present knowledge of early weights and measures comes from many sources. Some rather early standards have been recovered by archeologists and preserved in museums. The comparison of the dimensions of buildings with the descriptions of contemporary writers is another source of information. An interesting example of this is the comparison of the dimensions of the Greek Parthenon with the description given by Plutarch from which a fairly accurate idea of the size of the Attic foot is obtained. In some cases we have only plausible theories and we must sometimes decide on the interpretation to be given to the evidence. For example, does the fact that the length of the double-cubit of early Babylonia was equal (within two parts in a thousand) to the length of the seconds pendulum at Babylon indicate a scientific knowledge of the pendulum at a very early date, or do we merely have a curious coincidence? By studying the evidence given by all available sources, and by correlating the relevant facts, we obtain some idea of the origin and development of the units. We find that they have changed more or less gradually with the passing of time in a complex manner because of a great variety of modifying influences. We find the units modified and grouped into systems of weights and measures: The Babylonian system, the Phileterian system of the Ptolemaic age, the Olympic system of Greece, the Roman system, and the British system, to mention only a few.

3. Origin and development of some common units.- In the space available it will only be possible to give somewhat sketchily the story of the origin and development of a few common units.

One of the earliest units was the foot. This was first the length of the human foot without further specification or modification, then the length of the foot of various rulers of tribes and groups of people. Later, by gradual evolution, it was the foot as used in succession by the Egyptians, Greeks, and Romans, brought to Britain by the Romans, modified with the passing of time, and finally defined in Great Britain as $\frac{1}{3}$ of the British Imperial Yard and in this country as $\frac{1}{3}$ of the U. S. yard.

A very interesting and important unit of length used by many ancient peoples was the cubit, originally defined as the distance from the point of the elbow to the end of the middle finger. This unit was about 18 inches long, but there were important variations in the length of a cubit.

The inch was originally a thumb's breadth. In the Roman duodecimal system it was defined as $1/12$ foot, and was introduced into Britain during Roman occupation, where it became a part of the English system of weights and measures.

The mile was defined by the Romans as 1000 paces or double steps, the pace being equal to 5 Roman feet. This Roman mile of 5000 Roman feet was introduced into Britain, became 5000 English feet, and in Tudor times (probably in the reign of Henry VII, 1485 to 1509, but definitely by a statute of Queen Elizabeth, who reigned 1558 to 1603) was changed to 5280 feet in order to make the furlong of $1/8$ mile equal to the rod of 660 feet, or 220 yards (40 rods of $16\text{-}1/2$ feet or $5\text{-}1/2$ yards each).

The yard as a unit of length is apparently of much later origin than those previously discussed. It appears to have had a double origin; (1) as the length of an Anglo-Saxon gird or girdle, and (2) as the length of the double cubit. There is an old tradition, often stated as a fact, that Henry I decreed that the yard should thenceforth be the distance from the point of his nose to the end of his thumb.

Turning to units of weight, one of the earliest is the grain, which was originally the weight of a grain of wheat or of some specified seed native to some particular locality.

The Roman pound (libra) was the hundredth part of an older weight, the talent, which is believed to have been originally the weight of an Egyptian royal cubit of water. The Roman pound was divided into 12 ounces (unciae, meaning twelfth parts) of 437 grains each. This system was introduced into Britain where the pound was increased so as to have 16 of the original ounces. This pound became known as the avoirdupois pound. The idea of a pound divided into 16 parts was not a new one as the Greeks had divided their pound into 16 parts as well as into 12 parts. The pound which in England had long been used for mint purposes and called the troy pound consisted of 5760 grains (12 ounces of 480 grains each). The origin of this troy pound and troy ounce is very uncertain. One theory is that the troy pound came from Troyes, France, but there seems to be a serious question whether even the name had its origin in that place. Sometime prior to 1600 A.D., the avoirdupois pound was increased by 8 grains so that it would consist of 7000 grains instead of 6992 grains and thus the number of grains in the avoirdupois

pound would have a more simple ratio to the number of grains in the troy pound, which, being used for mint purposes, it was considered advisable to keep unchanged.

That the ton was the weight of a certain volume of some material is highly probable. Among the Anglo-Saxons it may have been the weight of a quantity of wheat in 32 bushels, that is, in one chaldron.

The stone was an early unit of weight in the British Isles. At one time it appears to have been 16 pounds in the system: 16 pounds = 1 stone, 16 stones = 1 wey, 16 weys = 1 last, and 1/2 last = 1 ton (not the present ton). The stone is a unit of weight still used to a considerable extent in Great Britain, being now equal to 14 pounds except in special cases. (3 stone = 1 cwt = 112 lb; 20 cwt = 1 ton = 2240 lb.)

A unit of antiquity which has survived without change is the degree of arc. The early Babylonians reckoned the year as 360 days. They therefore divided the circle into 360 parts or degrees. They knew that a chord equal to the radius subtends an arc of 60°. The number 60 became the basis of their sexagesimal number system and is an explanation of the division of the degree into 60 minutes and of the minute into 60 seconds. This is also the basis of the relation between longitude and time. Since the earth makes one complete rotation (360°) on its axis in 24 hours, a time change of 1 hour is represented by each 15° of longitude. ($360/24 = 15$)

II. The Metric System

1. The metric system: definition, origin and development.-- The metric system is the international decimal system of weights and measures based on the meter and the kilogram. The essential features of the system were embodied in a report made to the French National Assembly by the Paris Academy of Sciences in 1791. The definitive action taken in 1791 was the outgrowth of recommendations along similar lines dating back to 1670. The adoption of the system in France was slow, but its desirability as an international system was recognized by geodesists and others. On May 20, 1875, an international treaty was signed providing for an International Bureau of Weights and Measures, thus insuring "the international unification and improvement of the metric system."

The metric system is now either obligatory or permissive in every civilized country of the world.

Although the metric system is a decimal system, the words "metric" and "decimal" are not synonymous and care should be taken not to confuse the two terms. The metric system is not the same as the cgs system, and does not conform in all respects to the weights and measures laws and regulations of France.

2. Units and standards of the metric system.- In the metric system the fundamental units are the meter and the kilogram.¹ The other units of length and mass, as well as all units of area, volume, and capacity, also compound units, such as pressure, are derived from these two fundamental units.

The meter was originally intended to be one ten-millionth part of a meridional quadrant of the earth. The Meter of the Archives, the platinum end-standard which was the standard for most of the 19th century, at first was supposed to be exactly this fractional part of the quadrant. More refined measurements over the earth's surface showed that this supposition was not correct. The present international metric standard of length, the International Prototype Meter, a graduated line standard of platinum-iridium, was selected from a group of bars because it was found by precise measurements to have the same length as the Meter of the Archives. The meter is now defined as the distance under specified conditions between the lines on the International Prototype Meter without reference to any measurements of the earth or to the Meter of the Archives, which it superseded. In a similar manner the kilogram is now defined as the mass of the International Prototype Kilogram without reference to the mass of a cubic decimeter of water or to the Kilogram of the Archives. Each of the countries which subscribed to the International Metric Convention was assigned one or more copies of the international standards; these are known as National Prototype Meters and Kilograms.

The metric system, by itself, is not a complete system covering all physical measurements. A complete system requires certain additional units such, for example, as units of temperature and time.

3. The International Bureau of Weights and Measures.- The International Bureau of Weights and Measures was established at Sèvres, a suburb of Paris, France, by the International Metric Convention of May 20, 1875. At that Bureau there are kept the International Prototype Meter and the International Prototype Kilogram, many secondary standards of all sorts, and equipment for comparing standards and making precision measurements. This is an international bureau, not a French bureau, and is maintained by assessed contributions of the signatory governments.

¹The liter is a secondary or derived unit and is defined as the volume of a kilogram of pure water at the temperature of its maximum density and is equal to 1.000028 cubic decimeters according to the most recent determinations and computations. Formerly the value 1.000027 cubic decimeters was accepted as the best value.

In recent years the scope of the work at the International Bureau has been considerably broadened. It now carries on researches in the fields of electricity and photometry in addition to its former work in weights and measures with which was included such allied fields as thermometry and barometry.

4. Present status of the metric system in the United States.-- The use of the metric system in this country was legalized by Act of Congress in 1866. Its use has not been made obligatory except in a few special cases (see pages 8 and 9 of NBS Miscellaneous Publication M135, listed as No. 8 in the references on page 11 of this Letter Circular).

The United States Prototype Meter No. 27 and United States Prototype Kilogram No. 20 are recognized as the primary standards of length and mass of both the metric and the customary systems of measurement in this country, but this fact should not be taken as indicating that the metric system is regarded by the National Bureau of Standards as superior to the customary system of yards and pounds, but simply that these standards are regarded as the most precise and reliable standards available. Obviously it is not possible to accept both a meter and yard, and both a kilogram and a pound as "primary" standards, unless there is willingness to accept the possibility of continually changing the ratio between the corresponding units. In each case one must be accepted as the primary standard and the other derived therefrom by means of an accepted relation. In the United States the basic relations contained in the Law of July 28, 1866, which made the use of the metric system legal in the United States, and set forth in the Mendenhall Order* of April 5, 1893, are as follows:

$$\frac{1 \text{ U. S. yard}}{1 \text{ meter}} = \frac{3600}{3937}$$

and

$$\frac{1 \text{ U. S. pound}}{1 \text{ kilogram}} = \frac{10000}{22046}$$

A precise determination of the relation between the pound and the kilogram resulted in the value 1 pound = 0.4535924277 kilogram, and from which 1 kilogram = 2.204622341 pounds. In these two numbers more decimal places are retained than are justified by the precision attainable in the comparison of masses.

*This order stated that the Office of Weights and Measures with the approval of the Secretary of the Treasury, would in the future regard the International Prototype Meter and Kilogram as fundamental standards, and that the customary units would be derived therefrom, in accordance with the Act of July 28, 1866.

The official British relations between the corresponding units used in Great Britain are:

$$\frac{1 \text{ British yard}}{1 \text{ meter}} = \frac{36000000}{39370113}$$

and

$$1 \text{ British pound} = 0.45359243 \text{ kilogram}$$

In recent years intercomparisons between the British yard and the meter indicate that the official relation differs from the relation obtained by these intercomparisons by about a part in a million.

5. Arguments for and against the metric system.- That there are arguments both for and against the metric system is evidenced by the rather voluminous literature on this subject.

The National Bureau of Standards neither advocates nor opposes the compulsory adoption of the metric system. Those desiring arguments in favor of its compulsory adoption are referred to the Metric Association, Pottsville, Pennsylvania. Those desiring arguments against it are referred to the American Institute of Weights and Measures, 33 Rector Street, New York, N. Y. See also reference No. 7 in the Bibliography on page 11 of this Letter Circular.

III. British and United States Systems of Weights and Measures

The suggestion is sometimes made that the English system of weights and measures, instead of the metric system, should be adopted on an international basis. Aside from the fact that this would ignore the probable wishes of large numbers of peoples, the proposition disregards the fact that there are important differences between the system of weights and measures in general use in the British Empire and that in general use in the United States. It is true that the difference between the U. S. and the British inch is not significant except in a few cases of the most refined measurements, that the British and the U. S. pound may be considered identical, and that many tables such as 12 inches = 1 foot, 3 feet = 1 yard, and 1760 yards = 1 mile are the same in both countries. But there are some very important differences.

In the first place, the U. S. bushel, gallon, quart, and fluid ounce differ from the corresponding British units. Also the British ton is 2240 pounds, whereas the ton generally used in the United States is the short ton of 2000 pounds. The American colonists adopted the English wine gallon of 231 cubic inches. The English of that period used this wine gallon and they also had another gallon, the ale gallon of 282 cubic inches. In 1824 these two gallons were abandoned by the British when they adopted the British Imperial gallon, which is defined as the volume of 10 pounds of water, at a temperature of 62° F which, by calculation, is equivalent to 277.42 cubic inches.

At the same time the bushel was redefined as 8 gallons. In the British system the units of dry measure are the same as those of liquid measure. In the United States these two are not the same, the gallon and its subdivisions being used in the measurement of liquids, while the bushel, with its subdivisions, is used in the measurement of certain dry commodities. The U. S. gallon is divided into 4 liquid quarts and the U. S. bushel into 32 dry quarts. All the units of capacity mentioned thus far are larger in the British system than in the U. S. system. But the British quart is divided into 40 fluid ounces, while the U. S. quart is divided into 32 fluid ounces, and the British fluid ounce is smaller than the U. S. fluid ounce.

From the foregoing it is seen that in the British system an avoirdupois ounce of water at 62° F has a volume of 1 fluid ounce, since 10 pounds is equivalent to 160 avoirdupois ounces and 1 gallon is equivalent to 4 quarts, or 160 fluid ounces. This convenient relation does not obtain in the U. S. system since a U. S. gallon of water at 62° F weighs about 8-1/3 pounds, or 133-1/3 avoirdupois ounces, and the U. S. gallon is equivalent to 4 x 32 or 128 fluid ounces.

1 U. S. fluid ounce = 1.0408 British fluid ounces
1 British fluid ounce = 0.9608 U. S. fluid ounces

IV. Subdivision of Units

In general, units are subdivided in one of three ways, (a) decimally, that is successively to tenths; (b) duodecimally, to twelfths; or (c) binarily, to halves, quarters, eighths, etc. Each method has its advantages for certain purposes and it cannot properly be said that any one method is "best" unless the use to which the unit and its subdivisions are to be put is known.

For example, if we are concerned only with measurements of length to moderate precision, it is convenient to measure and to express these lengths in feet, inches, and binary fractions of an inch, thus 9 feet 4-3/8 inches. If, however, these measured lengths are to be subsequently used in calculations of area or volume, that method of subdivision at once becomes extremely inconvenient. For that reason civil engineers who are concerned with areas of land, volumes of cuts, fills, excavations, etc., instead of dividing the foot into inches and binary subdivisions of the inch, divide it decimally, that is, into tenths, hundredths, and thousandths of a foot.

On the other hand, machinists, toolmakers, gage makers, scientists and others who are engaged in precision measurements of relatively small distances, even though concerned with measurements of length only, find it convenient to use the inch, instead of the tenth of a foot, but to divide the inch decimally to tenths, hundredths, thousandths, etc., even down to millionths of an inch. Verniers, micrometers, and other precision measuring instruments are usually graduated in this manner. Machinists scales are commonly graduated decimally along one edge and are

also graduated along another edge to binary fractions as small as $1/64$ inch. The scales with binary fractions are used only for relatively rough measurements.

It is seldom convenient or advisable to use binary subdivisions of the inch that are smaller than $1/64$. In fact, $1/32$, $1/16$, or $1/8$ inch subdivisions are usually preferable for use on a scale to be read with the unaided eye.

The method of subdivision of a unit is thus largely made on the basis of convenience to the user. The fact that units have commonly been subdivided into certain sub-units for centuries does not preclude these units also having another mode of subdivision in some frequently used cases where convenience indicates the value of such other method. Thus the gallon is usually subdivided into quarts and pints, but the majority of gasoline measuring pumps of the price-computing type are graduated to show tenths of a gallon. Although the mile has for centuries been divided into rods, yards, feet, and inches, the odometer part of an automobile speedometer indicates tenths of a mile. Although our dollar is divided into 100 parts, we habitually use and speak of halves and quarters. An illustration of rather complex subdividing is found on the triangular scales used by draftsmen. These scales are of two types: (a) architects, which are commonly graduated with scales in which $3/32"$, $3/16"$, $1/8"$, $1/4"$, $3/8"$, $1/2"$, $3/4"$, $1"$, $1-1/2"$, and $3"$, respectively, represent 1 foot full scale, as well as having a scale graduated in the usual manner to $1/16"$; and (b) engineers, which are commonly subdivided to 10, 20, 30, 40, 50, and 60 parts to the inch.

The dictum of convenience applies not only to subdivisions of a unit but also to multiples of a unit. Elevations of land above sea level are given in feet even though the height may be several miles; the height of an airplane as given by an altimeter is likewise given in feet, no matter how high it may be.

V. Arithmetical Systems of Numbers

The subdivision of units of measurement is closely associated with arithmetical systems of numbers. The systems of weights and measures used in this country for commercial and scientific work, having many origins as has already been shown, naturally show traces of the various number systems associated with their origins and developments. Thus (a) the binary subdivision has come down to us from the Hindus, (b) the duodecimal system of fractions from the Romans, (c) the decimal system from the Chinese and Egyptians, some developments having been made by the Hindus, and (d) the sexagesimal system (division by 60), now illustrated in the subdivision of units of angle and of time, from the ancient Babylonians.

The suggestion is made from time to time that we should adopt a duodecimal number system and a duodecimal system of weights and measures. Another suggestion is for an octonary

number system (a system with 8 as the basis instead of 10 in our present system or 12 in the duodecimal) and an octonary system of weights and measures. Such suggestions have certain theoretical merits, but are very impractical because it is now too late to modify our number system and unwise to have arbitrary enforcement of any single system of weights and measures. It is far better for each branch of science, industry and commerce to be free to use whatever system has been found by experience best to suit its needs. The prime requisite of any system of weights and measures is that the units be definite. It is also important that the relations of these units to the units of other systems be definite, convenient and known, in order that conversion from one system to another may be accurately and conveniently made.

BIBLIOGRAPHY

Government publications marked thus (*) are not for free distribution, but can be obtained from the Superintendent of Documents, Government Printing Office, Washington, D. C., at the prices indicated (stamps not accepted). Those marked with the double asterisk (**) are available from the National Bureau of Standards, Washington, D. C., upon request. Many of the publications listed below will be found in some of the larger libraries.

1. William Hallock and Herbert T. Wade, "Outlines of the Evolution of Weights and Measures and the Metric System", Macmillan Co., New York, 1906. (Contains many references in the footnotes.)

2. Lowis D'A Jackson, "Modern Metrology", Crosby Lockwood & Co., London, 1882.

3. Edward Nicholson, "Men and Measures", Smith, Elder & Co., London, 1912.

4. G. Bigourdan, "Le Système Métrique", Gauthier-Villars et Cie., Paris, 1901.

5. Charles E. Guillaume, "La Convention du Mètre et le Bureau International des Poids et Mesures", Gauthier-Villars et Cie., Paris, 1902.

6. Charles E. Guillaume, "La Creation du Bureau International des Poids et Mesures et son Oeuvre", Gauthier-Villars et Cie., Paris, 1927.

7. Julia Emily Johnsen, (Compiler), "Metric System", The H. H. Wilson Co., New York, 1926. (Contains bibliography, material useful for debates, etc.)

*8. National Bureau of Standards Miscellaneous Publication M 135, "The International Metric System of Weights and Measures", (supersedes Miscellaneous Publication M2), 5 cents.

*9. National Bureau of Standards Miscellaneous Publication M3, "The International Metric System", 30 cents. (A chart in colors, 28-1/2 by 44 inches, giving graphic comparisons between the units of the metric system and the customary system of weights and measures.)

*10. National Bureau of Standards Miscellaneous Publication M64, "History of the Standard Weights and Measures of the United States", by Louis A. Fischer (illustrated), 15 cents.

*11. National Bureau of Standards Miscellaneous Publication M121, "Units of Weight and Measure, (United States and Metric) Definitions and Tables of Equivalents", (Supersedes Circular C47). 15 cents.

*12. National Bureau of Standards Miscellaneous Publication M122, "Weights and Measures in Congress", by Sarah Ann Jones, 5 cents.

**13. National Bureau of Standards Letter Circular LC 376, "Metric and English Distance Equivalents for Athletic Events (track and field)."

**14. National Bureau of Standards Letter Circular LC 449, "Standards of Length, Mass, and Time."

*15. National Bureau of Standards Letter Circular LC 517, "Motorists' Manual of Weights and Measures."

**16. Henry W. Bearce, "United States and British Units of Weights and Measures", Scientific Monthly, v. XLIII, pp. 566 - 568 (July 1936).

**17. National Bureau of Standards Letter Circular LC 682, "General Tables of Weights and Measures."

Considerable information on weights and measures will be found in such encyclopedias as Encyclopedia Britannica and in such dictionaries as Webster's New International.